Department for Knowledge and Language Engineering
Computational Intelligence
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Final Examination in "Fuzzy Systems"

last name, surname:	faculty:	studies:	matricul. no.:
type of exam: ☐ 1st/2nd trial ☐ ungraded certificate ☐ graded certificate	signature of supervision:		no. sheets:

Asgmt. 1	Asgmt. 2	Asgmt. 3	Asgmt. 4	Asgmt. 5	Total
/11	/5+1	/16	/10+1	/8	/50+2

Assignment 1 Generator Functions (11 points, ca. 25 minutes)

One possibility to determine fuzzy negations, t-norms and t-conorms is the concept of generator functions. An increasing generator function is a continuous and strictly monotonically increasing function $g : [0, 1] \to \mathbb{R}$ where $g(0) = 0$. Let g^{-1} be the inverse of g. Let the pseudo-inverse of g be defined by

$$g^{(-1)}(a) = \begin{cases} 0 & \text{if } a < 0, \\ g^{-1}(a) & \text{if } 0 \le a \le g(1), \\ 1 & \text{if } a > g(1). \end{cases}$$

With such a generator function, we can compute an involutive fuzzy negation by

$$\sim a = g^{-1}(g(1) - g(a)),$$

an Archimedian t-norm by

$$\top(a, b) = g^{(-1)}(g(a) + g(b) - g(1)),$$

an Archimedian t-conorm by

$$\bot(a, b) = g^{(-1)}(g(a) + g(b)).$$

Now, consider the increasing generator function

$$g(a) = \frac{2a^2}{3}$$

and determine its induced triple of fuzzy negation, t-norm and t-conorm.

Assignment 2 Fuzzy Relational Equations (5 + 1 points, ca. 10 minutes)

Let $X = \{a, b\}$ and $Y = \{p, q, r\}$. Consider the fuzzy sets μ_1, μ_2, μ_3 on X and ν_1, ν_2, ν_3 on Y which are defined as shown below.

	a	b
μ_1	0.0	0.3
μ_2	0.6	0.4
μ_3	0.8	1.0

	p	q	r
ν_1	0.2	0.7	0.6
ν_2	0.3	0.5	0.6
ν_3	0.3	0.7	0.8

a) Find the greatest solution of the system of fuzzy relational equations $\mu_i \circ \varrho = \nu_i$ for $i = 1, 2, 3$.

b) *Extra:* Find one smallest solution of this system.

Assignment 3 Fuzzy Arithmetic (16 points, ca. 40 minutes)

Consider the following two fuzzy numbers:

$$\mu_1(x) = \begin{cases} 1 + x & \text{if } -1 \le x \le 0, \\ 1 - x & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_2(x) = \begin{cases} x & \text{if } 0 \le x \le 1, \\ 2 - x & \text{if } 1 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the fuzzy sets $\mu_1 + \mu_2$, $\mu_1 - \mu_2$, and $\mu_1 \cdot \mu_2$ using the set representation of the fuzzy numbers.

Hint: Computing the inverse of a quadratic function $f(x) = ax^2 + bx + c$, its vertex form $f(x) = a(x - h)^2 + k$ might be useful where (h, k) is the vertex of f.

Assignment 4 Mamdani-Assilian Control (10 + 1 points, ca. 25 minutes)

Design a Mamdani-Assilian controller for the operation of an unmanned hot-air balloon. The goal is to keep the balloon at constant altitude. This can be performed by controlling both an outlet valve and the burner flame. The balloon is equipped with sensors which determine the (signed) deviations of target altitude and current vertical speed.

a) Define linguistic variables to describe the necessary variables by choosing appropriate fuzzy sets. Draw every linguistic variable in a draft.

b) Suggest operations to compute the rule activations, to combine outputs of simultaneously active rules, and to defuzzify the final output fuzzy set.

c) Specify a set of rules that is suitable to control the balloon.

   Extra: Which conditions should be satisfied regarding the control variables?

Assignment 5 Takagi-Sugeno-Kang Control (8 points, ca. 20 minutes)

Determine a Takagi-Sugeno-Kang controller with one input and one output that approximates the function shown below for inputs from the interval $[0, 10]$.