Fuzzy Systems
Prof. Dr. Rudolf Kruse, Christian Moewes

Assignment Sheet 11

Assignment 41 Similarity Relations

In the lecture an similarity relation was defined as follows:

Definition: *A function* $E : X \times X \to [0, 1]$ *is called an similarity relation if it satisfies for all* $x, y, z \in X$

(i) $E(x, x) = 1$,

(ii) $E(x, y) = E(y, x)$ *and*

(iii) $\max\{E(x, y) + E(y, z) - 1, 0\} \leq E(x, z)$.

This is only a simplified definition which is based on the t-norm $\top_{\text{Łuka}}(a, b) = \max\{a+b-1, 0\}$, *i.e.* the most commonly used t-norm to define similarity relations. In general, any t-norm may be used in the third condition (which, by the way, prevents the occurrence of Poincaré's paradox). Suppose now that the third condition of the above definition is replaced by

(iii) $\top_{\min}(E(x, y), E(y, z)) = \min\{E(x, y), E(y, z)\} \leq E(x, z)$.

Find at least two similarity relations that satisfy this modified definition. Hint: It may be easier to consider domains with a finite number of values.

Assignment 42 Similarity Relations

In the lecture the notion of an extensional hull of a set was introduced as follows:

Definition: *Let* E *be an similarity relation on a set* X*. Then the extensional hull of a set* $M \subseteq X$ *is the fuzzy set*

$$\mu_M : X \to [0, 1], \qquad x \mapsto \sup\{E(x, y) \mid y \in M\}.$$

Determine the extensional hulls of the sets $\{2\}$, $\{3, 4\}$ and $[3, 4]$ *w.r.t.* the similarity relation $E(x, y) = 1 - \min\{|x - y|,\ 1\}$. How do these extensional hulls relate to fuzzy sets?

Assignment 43 Similarity Relations

Reconsider the Mamdani-Assilian controller studied in Assignment 34. This controller had two inputs $\xi_1 \in X_1 = [-1, 1]$ and $\xi_2 \in X_2 = [-1, 1]$ and one output $\eta \in Y = [-1, 1]$. The fuzzy partitions are shown again below on the left ("az" means "approximately zero"). The rule base of the controller is shown on the right in tabular form.

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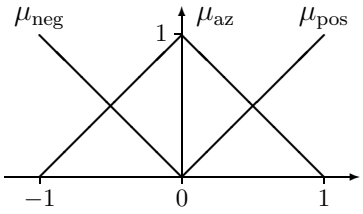

ξ_2 \ ξ_1	neg	az	pos
neg	neg		az
az		az	
pos	az		pos

Find an interpretation of this fuzzy controller on the basis of similarity equations. That is, find (precise) input/output pairs and an similarity relation that yields the above Mamdani-Assilian controller.